

# Incremental Integer Linear Programming for Non-projective Dependency Parsing

Sebastian Riedel James Clarke

ICCS, University of Edinburgh

22. July 2006  
EMNLP 2006

# Labelled Dependency Parsing

## Labelled Dependency Parsing

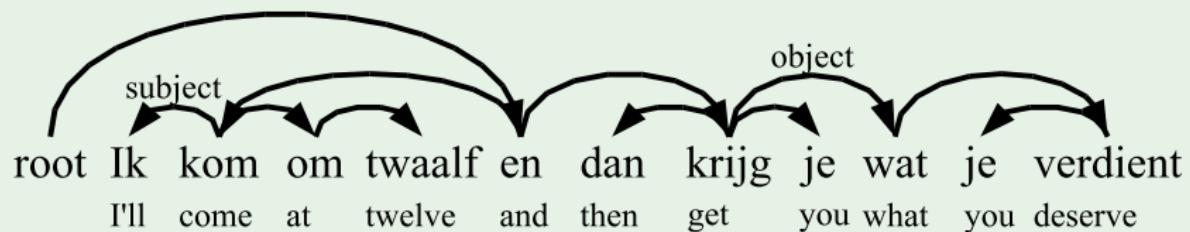
Find labelled head-child relations between tokens.

# Labelled Dependency Parsing

## Labelled Dependency Parsing

Find labelled head-child relations between tokens.

### Example



# Non-projective Dependency Parsing

## Non-projective Dependency Parsing

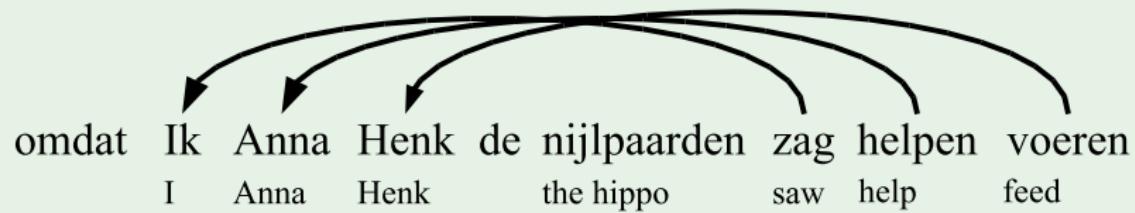
Dependencies are allowed to cross

# Non-projective Dependency Parsing

## Non-projective Dependency Parsing

Dependencies are allowed to cross

### Example



# Non-projective Dependency Parsing

## Non-projective Dependency Parsing

Dependencies are allowed to cross

### Example



### Methods

- Nivre et al. (2004)
- Yamada and Matsumoto (2003)
- McDonald et al. (2005)

# McDonald et al. (2005)

## McDonald et al. (2005)

- State-of-the-art non-projective dependency parser.
- Based on finding the maximum spanning tree.
- Attachment decisions made **independently**.

# McDonald et al. (2005)

## McDonald et al. (2005)

- State-of-the-art non-projective dependency parser.
- Based on finding the maximum spanning tree.
- Attachment decisions made **independently**.

## Example Mistake on Alpino Corpus

root Ik kom om twaalf en dan krijg je wat je verdient  
I'll come at twelve and then get you what you deserve



# McDonald et al. (2005)

## McDonald et al. (2005)

- State-of-the-art non-projective dependency parser.
- Based on finding the maximum spanning tree.
- Attachment decisions made **independently**.

## Example Mistake on Alpino Corpus

root Ik kom om twaalf en dan krijg je wat je verdient  
I'll come at twelve and then get you what you deserve



## McDonald and Pereira, 2006

- Second order scores
- *Approximate* search

# More General

Chu-Liu-Edmonds, CYK, Viterbi

- More local models
- Optimality guaranteed
- Polynomial runtime guaranteed

# More General

## Chu-Liu-Edmonds, CYK, Viterbi

- More local models
- Optimality guaranteed
- Polynomial runtime guaranteed

## Beam Search, Sampling

- More global models
- Optimality not guaranteed
- Polynomial runtime guaranteed

# More General

## Chu-Liu-Edmonds, CYK, Viterbi

- More local models
- Optimality guaranteed
- Polynomial runtime guaranteed

## Beam Search, Sampling

- More global models
- Optimality not guaranteed
- Polynomial runtime guaranteed

## Incremental ILP

- More global models
- Optimality guaranteed
- Polynomial runtime not guaranteed

# Overview

1 Maximum Spanning Tree Problem

2 Linguistic Constraints

3 Decoding

- Decoding with Integer Linear Programming(ILP)
- Incremental ILP
- Parsing Example

4 Training

5 Experiments

6 Conclusion

# Outline

## 1 Maximum Spanning Tree Problem

## 2 Linguistic Constraints

## 3 Decoding

- Decoding with Integer Linear Programming(ILP)
- Incremental ILP
- Parsing Example

## 4 Training

## 5 Experiments

## 6 Conclusion

# Maximum Spanning Tree Problem

## Example Graph with Scores



## MST Objective

Find the tree with the maximum sum of scores

# More Formal

## Score

$$s(\mathbf{x}, \mathbf{y}) = \sum_{(i,j,l) \in \mathbf{y}} s(i,j,l) = \sum_{(i,j,l) \in \mathbf{y}} \mathbf{w} \cdot \mathbf{f}(i,j,l)$$

for graph  $\mathbf{y}$

# More Formal

Score

$$s(\mathbf{x}, \mathbf{y}) = \sum_{(i,j,l) \in \mathbf{y}} s(i,j,l) = \sum_{(i,j,l) \in \mathbf{y}} \mathbf{w} \cdot \mathbf{f}(i,j,l)$$

for graph  $\mathbf{y}$

Constraint (Exactly One Head)

Exactly one head for each non-root token; no head for root

# More Formal

Score

$$s(\mathbf{x}, \mathbf{y}) = \sum_{(i,j,l) \in \mathbf{y}} s(i,j,l) = \sum_{(i,j,l) \in \mathbf{y}} \mathbf{w} \cdot \mathbf{f}(i,j,l)$$

for graph  $\mathbf{y}$

Constraint (Exactly One Head)

Exactly one head for each non-root token; no head for root

Constraint (No Cycles)

The dependency graph contains no cycles

# More Formal

Score

$$s(\mathbf{x}, \mathbf{y}) = \sum_{(i,j,l) \in \mathbf{y}} s(i,j,l) = \sum_{(i,j,l) \in \mathbf{y}} \mathbf{w} \cdot \mathbf{f}(i,j,l)$$

for graph  $\mathbf{y}$

Constraint (Exactly One Head)

Exactly one head for each non-root token; no head for root

Constraint (No Cycles)

The dependency graph contains no cycles

MST Objective

Maximise score under the two above constraints

# Outline

1 Maximum Spanning Tree Problem

2 Linguistic Constraints

3 Decoding

- Decoding with Integer Linear Programming(ILP)
- Incremental ILP
- Parsing Example

4 Training

5 Experiments

6 Conclusion

# Linguistic Constraints

## Constraint

Coordination arguments must be compatible

## Violated in

root Ik kom om twaalf en dan krijg je wat je verdient  
I'll come at twelve and then get you what you deserve



# Linguistic Constraints

## Constraint

There must not be more than one subject for each verb

## Violated in

subject      subject

root Ik kom om twaalf en dan krijg je wat je verdient  
I'll come at twelve and then get you what you deserve

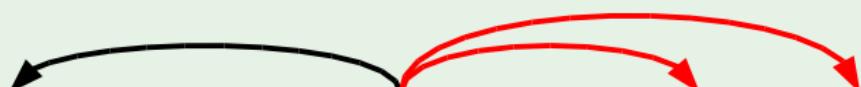
# Linguistic Constraints

## Constraint

For each *and* coordination there is exactly one argument to the right and one more arguments to the left

## Violated in

root Ik kom om twaalf en dan krijg je wat je verdient  
I'll come at twelve and then get you what you deserve



# Outline

1 Maximum Spanning Tree Problem

2 Linguistic Constraints

3 Decoding

- Decoding with Integer Linear Programming(ILP)
- Incremental ILP
- Parsing Example

4 Training

5 Experiments

6 Conclusion

# Outline

1 Maximum Spanning Tree Problem

2 Linguistic Constraints

3 Decoding

- Decoding with Integer Linear Programming(ILP)
- Incremental ILP
- Parsing Example

4 Training

5 Experiments

6 Conclusion

# Decoding

## Objective

Maximise:

$$s(\mathbf{x}, \mathbf{y}) = \sum_{(i,j,l) \in \mathbf{y}} s(i, j, l)$$

given

- dependency parsing constraints
- linguistic constraints

# Decoding

## Objective

Maximise:

$$s(\mathbf{x}, \mathbf{y}) = \sum_{(i,j,l) \in \mathbf{y}} s(i, j, l)$$

given

- dependency parsing constraints
- linguistic constraints

## Methods

- Use the Chu-Liu-Edmonds algorithm (McDonald et al., 2005)
- Use some approximate search (McDonald and Pereira, 2006)
- Use Integer Linear Programming (Roth and Yih, 2005)

# Decoding

## Objective

Maximise:

$$s(\mathbf{x}, \mathbf{y}) = \sum_{(i,j,l) \in \mathbf{y}} s(i, j, l)$$

given

- dependency parsing constraints
- linguistic constraints

## Methods

- Use the Chu-Liu-Edmonds algorithm (McDonald et al., 2005)
- Use some approximate search (McDonald and Pereira, 2006)
- Use Integer Linear Programming (Roth and Yih, 2005)

# Decoding

## Objective

Maximise:

$$s(\mathbf{x}, \mathbf{y}) = \sum_{(i,j,l) \in \mathbf{y}} s(i, j, l)$$

given

- dependency parsing constraints
- linguistic constraints

## Methods

- Use the Chu-Liu-Edmonds algorithm (McDonald et al., 2005)
- Use some approximate search (McDonald and Pereira, 2006)
- Use Integer Linear Programming (Roth and Yih, 2005)

# Integer Linear Programming (ILP)

## Decision Variables

$x_1, x_2, x_3$

# Integer Linear Programming (ILP)

## Decision Variables

$x_1, x_2, x_3$

## Objective Function

$$1.5x_1 + 2x_2 - x_3$$

# Integer Linear Programming (ILP)

## Decision Variables

$$x_1, x_2, x_3$$

## Objective Function

$$1.5x_1 + 2x_2 - x_3$$

## Linear Constraints

$$x_1 + x_2 < 2$$

$$x_1 - x_3 > 1$$

# Integer Linear Programming (ILP)

## Decision Variables

$$x_1, x_2, x_3$$

## Linear Constraints

$$\begin{aligned}x_1 + x_2 &< 2 \\x_1 - x_3 &> 1\end{aligned}$$

## Objective Function

$$1.5x_1 + 2x_2 - x_3$$

## Integer Constraints

$$x_1 \in \{0, 1\}$$

# Integer Linear Programming (ILP)

## Decision Variables

$$x_1, x_2, x_3$$

## Linear Constraints

$$\begin{aligned}x_1 + x_2 &< 2 \\x_1 - x_3 &> 1\end{aligned}$$

## Objective Function

$$1.5x_1 + 2x_2 - x_3$$

## Integer Constraints

$$x_1 \in \{0, 1\}$$

## ILP Objective

Maximise objective function under constraints.

# Integer Linear Programming (ILP)

## Decision Variables

$$x_1, x_2, x_3$$

## Linear Constraints

$$\begin{aligned}x_1 + x_2 &< 2 \\x_1 - x_3 &> 1\end{aligned}$$

## Objective Function

$$1.5x_1 + 2x_2 - x_3$$

## Integer Constraints

$$x_1 \in \{0, 1\}$$

## ILP Objective

Maximise objective function under constraints.

## Taskar 2004

Every Markov Network can be mapped to an polynomial-size ILP.

# Dependency Parsing with ILP

## Decision Variables

$$e_{i,j,l} = \begin{cases} 1 & \text{if there is a dependency from } i \text{ to } j \text{ with label } l \\ 0 & \text{otherwise} \end{cases}$$

for each token  $i, j$  and label  $l$

## Objective Function

$$\sum_{i,j,l} s(i,j,l) \cdot e_{i,j,l}$$

# Dependency Parsing with ILP (2)

## Auxiliary Variables

$$d_{i,j} = \begin{cases} 1 & \text{if there is a dependency from } i \text{ to } j \\ 0 & \text{otherwise} \end{cases}$$

for each token  $i, j$

# Dependency Parsing with ILP (2)

## Auxiliary Variables

$$d_{i,j} = \begin{cases} 1 & \text{if there is a dependency from } i \text{ to } j \\ 0 & \text{otherwise} \end{cases}$$

for each token  $i, j$

## Only One Head

$$\sum_i d_{i,j} = 1$$

for all  $j > 0$ .

# Dependency Parsing with ILP (2)

## Auxiliary Variables

$$d_{i,j} = \begin{cases} 1 & \text{if there is a dependency from } i \text{ to } j \\ 0 & \text{otherwise} \end{cases}$$

for each token  $i, j$

## Only One Head

$$\sum_i d_{i,j} = 1$$

for all  $j > 0$ .

## No Cycles

$$\sum_{(i,j) \in G_s} d_{i,j} \leq |s|$$

for possible subsets all sets  $s$  of tokens

# Dependency Parsing with ILP (2)

## Auxiliary Variables

$$d_{i,j} = \begin{cases} 1 & \text{if there is a dependency from } i \text{ to } j \\ 0 & \text{otherwise} \end{cases}$$

for each token  $i, j$

## Only One Head

$$\sum_i d_{i,j} = 1$$

for all  $j > 0$ .

## No Cycles

$$\sum_{(i,j) \in G_s} d_{i,j} \leq |s|$$

for possible subsets **all sets**  $s$  of tokens

# Dependency Parsing with ILP (2)

## Auxiliary Variables

$$d_{i,j} = \begin{cases} 1 & \text{if there is a dependency from } i \text{ to } j \\ 0 & \text{otherwise} \end{cases}$$

for each token  $i, j$

## Only One Head

$$\sum_i d_{i,j} = 1$$

for all  $j > 0$ .

## No Cycles

$$\sum_{(i,j) \in G_s} d_{i,j} \leq |s|$$

for possible subsets **all sets**  $s$  of tokens

Germann et al. (2001)

Same cycle problem in ILP formulation for MT

# Outline

1 Maximum Spanning Tree Problem

2 Linguistic Constraints

3 Decoding

- Decoding with Integer Linear Programming(ILP)
- **Incremental ILP**
- Parsing Example

4 Training

5 Experiments

6 Conclusion

# Incremental Integer Linear Programming

## Setup

- *base* (e.g. exactly one head) constraints
- *incremental* (e.g. no cycles) constraints

# Incremental Integer Linear Programming

## Setup

- *base* (e.g. exactly one head) constraints
- *incremental* (e.g. no cycles) constraints

## Algorithm (see Warme (2002))

Set up ILP I with objective function and *base* constraints

**repeat**

Solve I

Find violated *incremental* constraints

Add constraints to I

**until** No more constraints violated

# Incremental Integer Linear Programming

## Setup

- *base* (e.g. exactly one head) constraints
- *incremental* (e.g. no cycles) constraints

## Algorithm (see Warme (2002))

Set up ILP I with objective function and *base* constraints

**repeat**

Solve I

Find violated *incremental* constraints

Add constraints to I

**until** No more constraints violated

## Tromble and Eisner (2006)

Replace ILP with finite-state automata

# Outline

1 Maximum Spanning Tree Problem

2 Linguistic Constraints

3 Decoding

- Decoding with Integer Linear Programming(ILP)
- Incremental ILP
- Parsing Example

4 Training

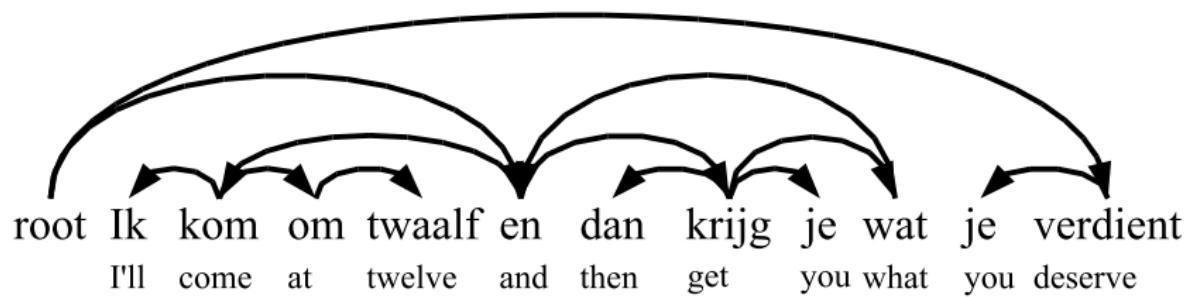
5 Experiments

6 Conclusion

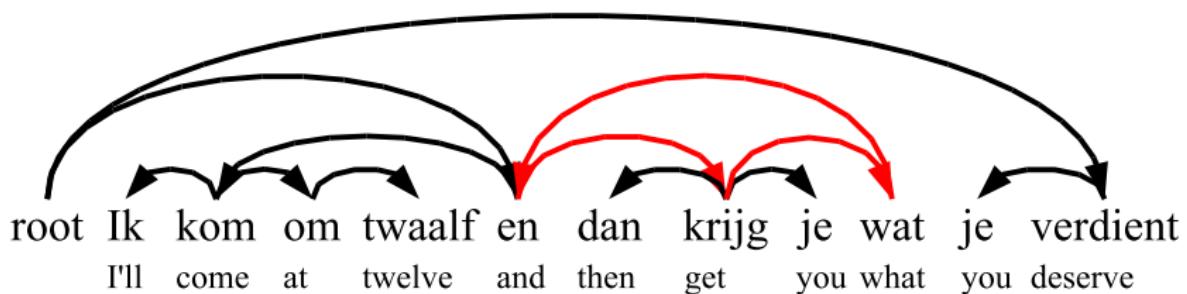
# Example Sentence

root Ik kom om twaalf en dan krijg je wat je verdient  
I'll come at twelve and then get you what you deserve

## First Solution



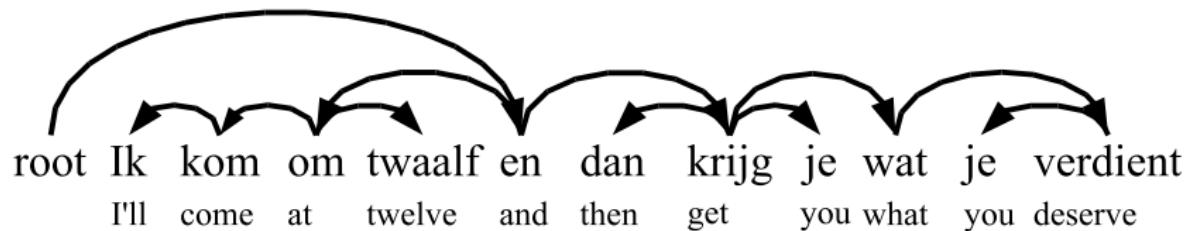
# Add Violated Constraints



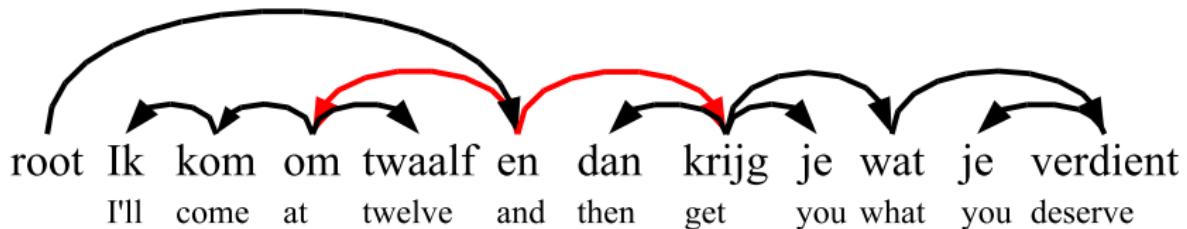
Add Constraint

$$d_{what, and} + d_{and, get} + d_{get, what} < 3$$

## Next Solution



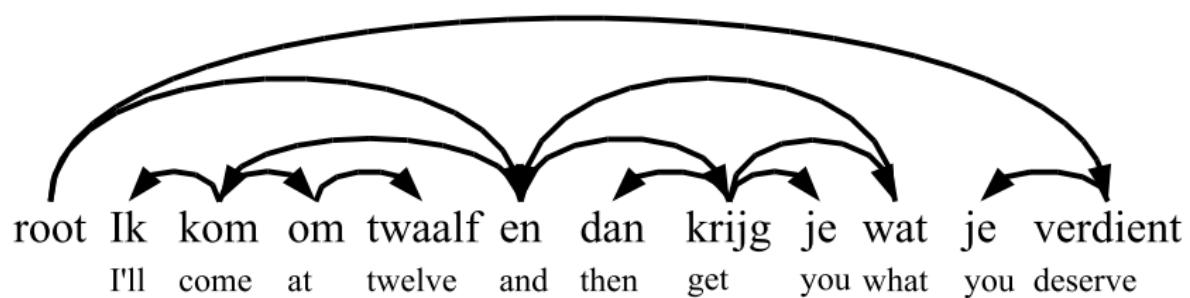
# Add Violated Constraints



Add Constraint

$$d_{and,at} + d_{and,get} < 2$$

Done



# Outline

1 Maximum Spanning Tree Problem

2 Linguistic Constraints

3 Decoding

- Decoding with Integer Linear Programming(ILP)
- Incremental ILP
- Parsing Example

4 Training

5 Experiments

6 Conclusion

# Training

## Online Learning

- ① single-best MIRA
- ② Chu-Liu-Edmonds for parsing (McDonald et al. 2005)
- ③ No constraints

# Training

## Online Learning

- ① single-best MIRA
- ② Chu-Liu-Edmonds for parsing (McDonald et al. 2005)
- ③ No constraints

Roth and Yih (2005)

Training without constraints can actually help

# Outline

1 Maximum Spanning Tree Problem

2 Linguistic Constraints

3 Decoding

- Decoding with Integer Linear Programming(ILP)
- Incremental ILP
- Parsing Example

4 Training

5 Experiments

6 Conclusion

# Experiments

## Questions

- How accurate in comparison to McDonald et. al (2005)?
- How fast/slow?

# Experiments

## Questions

- How accurate in comparison to McDonald et. al (2005)?
- How fast/slow?

## Data

- Dutch alpino corpus from the CoNLL shared task 2006
- about 13000 Sentences, non-projective, 5% of edges crossing
- Split into development set and crossvalidation set
- Tuned feature and constraint set on dev set

# Accuracy

In Comparison with McDonald et. al (2005)

| Crossvalidation | Labelled | Unlabelled | Complete | Complete(U) |
|-----------------|----------|------------|----------|-------------|
| McDonald 2005   | 84.6%    | 88.9%      | 27.7%    | 42.2%       |
| Incremental ILP | 85.1%    | 89.4%      | 29.7%    | 43.8%       |

Statistical significant ( $p < 0.001$  for Sign test and Dan Bikel's parse eval script)

# Accuracy

In Comparison with McDonald et. al (2005)

| Crossvalidation | Labelled | Unlabelled | Complete | Complete(U) |
|-----------------|----------|------------|----------|-------------|
| McDonald 2005   | 84.6%    | 88.9%      | 27.7%    | 42.2%       |
| Incremental ILP | 85.1%    | 89.4%      | 29.7%    | 43.8%       |

Statistical significant ( $p < 0.001$  for Sign test and Dan Bikel's parse eval script)

## With Others

- Wins on CoNLL test set but not significantly better than McDonald et al. (2006)
- Similar to performance of Malouf and van Noord (2004) (84.4% , smaller training set, evaluates control relations)

# Runtime Evaluation

## Exact Inference

- reasonable fast (0.5s for sentences with length between 20 - 30 tokens)
- significantly slower than McDonald et al. (2005) (3ms!)
- 150 times slower when parsing the full corpus (50min vs 20s) without feature extraction
- 6 times slower with feature extraction (add 10 minutes)
- 2 times slower with nearly loss less approximation method (see paper)

# Outline

1 Maximum Spanning Tree Problem

2 Linguistic Constraints

3 Decoding

- Decoding with Integer Linear Programming(ILP)
- Incremental ILP
- Parsing Example

4 Training

5 Experiments

6 Conclusion

# Conclusion

## Accuracy

- Significantly better than McDonald et. al (2005)
- headroom left - rule engineering

# Conclusion

## Accuracy

- Significantly better than McDonald et. al (2005)
- headroom left - rule engineering

## Runtime

- Significantly slower than McDonald et. al (2005)
- Feature extraction dominates runtime
- almost loss-less approximation available

# Conclusion

## Accuracy

- Significantly better than McDonald et. al (2005)
- headroom left - rule engineering

## Runtime

- Significantly slower than McDonald et. al (2005)
- Feature extraction dominates runtime
- almost loss-less approximation available

## In General

- Allows global models
- Guarantees optimality
- No polynomial runtime guarantee
- Good scores - fast processing

# Future Work

## Parsing

- 2nd order features
- Evaluate on more languages
- Joint POS tagging and parsing
- Joint constituent and dependency parsing

## General

- Other applications(Collective IE, MT?)
- Generalise to features/potentials (as opposed to constraints)
- Theoretical runtime estimates

Thank you